Non-local SFT tachyon and cosmology

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ABSTRACT: Cosmological scenarios built upon the generalized non-local String Field Theory and p-adic tachyons are examined. A general kinetic operator involving an infinite number of derivatives is studied as well as arbitrary parameter p. The late time dynamics of just the tachyon around the non-perturbative vacuum is shown to leave the cosmology trivial. A late time behavior of the tachyon and the scale factor of the FRW metric in the presence of the cosmological constant or a perfect fluid with $w > -1$ is constructed explicitly and a possibility of non-vanishing oscillations of the total effective state parameter around the phantom divide is proven.

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1. Introduction

Contemporary cosmological observational data [1] strongly support that the present Universe exhibits an accelerated expansion providing thereby an evidence for a dominating Dark Energy (DE) component. Recent results of WMAP [2] together with the data on Ia supernovae give the following bounds for the DE state parameter:

$$w_{\mathrm{DE}} = -0.97^{+0.07}_{-0.09}$$

or without an a priori assumption that the Universe is flat and together with the data on large-scale structure and supernovae $w_{\mathrm{DE}} = -1.06^{+0.13}_{-0.08}$.

The phantom divide $w = -1$ separates the quintessence models, $w > -1$ [3, 4], containing an extra light scalar field which is not in the Standard Model set of fields [5], the cosmological constant, $w = -1$ [6, 7], and the "phantom" models, $w < -1$, which can be realized by a scalar field with a ghost (phantom) kinetic term. In this case all natural energy conditions are violated and there are problems of instability both at the classical and quantum levels [8, 9].

Experimental data, as we see, do not contradict a possibility $w < -1$ and moreover the direct search strategy to test inequality $w < -1$ has been proposed [10]. Studying of such models attracts a lot of attention. Some projects [11] explore whether w varies with the

time or is an exact constant. Varying w obviously corresponds to a dynamical model of the DE which generally speaking includes a scalar field. Modified models of General Relativity also generate an effective scalar field (see for example [12] and refs. therein). Other DE models based on brane-world scenarios are presented in [13]. An excellent review [14] and references therein may provide the reader with a more detailed discussion of the DE dynamics.

Models with a crossing of the $w = -1$ barrier are also a subject of recent studies. Simplest ones include two scalar fields (one phantom and one usual field, see [15, 16] and refs. therein). General κ-essence models [17, 18] can have both $w < -1$ and $w \geqslant -1$ but a dynamical transition between these domains is forbidden under general assumptions [19] and is possible only under special conditions [20].

In the present paper we investigate cosmological applications of a scalar field model coming from the String Field Theory (SFT) tachyon dynamics (see [21] for a review) described by the action

$$ S = \frac{1}{g_o^2} \int d^d x \left(\frac{1}{2} \Phi \mathcal{F}(\Box) \Phi - \frac{1}{p+1} \Phi^{p+1}(x) \right). $$

Here g_o is a coupling constant. Function $\mathcal{F}(\Box)$ is not specified explicitly. Such a theory describes in particular an effective open SFT tachyon as well as a p-adic formulation [22, 23] of the tachyon dynamics. In both these examples a kinetic operator $\mathcal{F}$ gives a non-local action. Analysis of the Dirac-Born-Infeld approach to the tachyon cosmology may be found in [24, 25] and references therein.

The cosmological model incorporating the non-local dynamics of the open SFT tachyon field proposed in [26]. This model is based on the SFT formulation of the fermionic NSR string with the GSO$-$ sector [27] and its cosmological applications were studied in [28–30]. A characteristic feature of this model in the flat background is a presence of a rolling tachyon solution [31, 32].[1] In the bosonic SFT, however, such a solution does not exist [34, 35] at least in the flat space. The dynamics of a non-local tachyon on a cosmological background in the Hamilton-Jacobi formalism is studied in [36]. It is explicitly shown in [29] that the non-locality provides a crossing of the $w = -1$ barrier in spite of the presence of only one scalar field and being a string theory limit the model addresses all stability issues [37] to the string theory.[2] In a recent paper [39] p-adic inflationary models are constructed using similar non-local Lagrangian.[3]

Our present goal is a general analysis without specifying an exact form of operator $\mathcal{F}$ and a qualitative analysis of linearized Friedmann equations in this model at large times. From the point of view of the SFT such a generalization is natural since form of the kinetic operator depends on the contents of string excitations taken into account as well as parameter p will not be necessarily 3 as it is in the above cited papers. It is shown in the current paper that the form of operator $\mathcal{F}$ is the most crucial ingredient. The only

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[1] Other works which address an issue of solving the SFT equations of motion for homogenous time-dependent tachyon profiles are, for instance, [33] and refs. therein.

[2] In [38] it has been proposed a phantom model without an UV pathology in which a vector field is used.

[3] See also [40] where a non-local modification of gravity is studied.

physical assumption made is that a non-perturbative vacuum does exist and there are no open string excitations in it since being associated with the open string tachyon, our scalar field Φ describes according to Sen's conjecture [41] a transition of an unstable brane to the true vacuum where no perturbative states present.

The paper is organized as follows. In section 2 we set up the model and write down the Friedmann equations. In section 3 we explore how the non-local action for the scalar field linearized near a non-perturbative vacuum can be rewritten in a local form. In section 4 we compute cosmological quantities for a general operator $\mathcal{F}$ and an arbitrary background as well as formulate a perturbative approach for solving the Friedmann equations. In section 5 we consider the cosmological dynamics of just the tachyon in the initially flat space, i.e. $H(t_0) = 0$ and show that nothing can be generated dynamically. In section 6 we analyze two specific cosmological backgrounds keeping operator $\mathcal{F}$ arbitrary. In the last section the obtained results are discussed.

2. Model set-up

The action for the tachyon in the Cubic Super SFT [42, 43] in the flat background[4] when fields up to zero mass are taken into account is found to be [27, 31]

$$S_\Phi = \frac{1}{g_o^2} \int d^d x \left(\frac{1}{2} \Phi \mathcal{F}(\Box) \Phi - \frac{1}{4} \Phi^4 \right) \tag{2.1}$$

with

$$\mathcal{F}(\omega^2) = \left(\xi^2 \omega^2 + 1 \right) e^{-\frac{1}{4}\omega^2}. \tag{2.2}$$

where $\xi^2 \approx 0.9556$ is a constant entirely determined by SFT and we put $\alpha' = 1$. Scalar field Φ is the open string tachyon in question. This action is obviously a non-local one since it contains an infinite number of derivatives. Further it may be interesting to generalize the latter expression to bring it to a p-adic string like form by replacing 4 with $p + 1$ as follows

$$S = \frac{1}{g_o^2} \int d^d x \left(\frac{1}{2} \Phi \mathcal{F}(\Box) \Phi - \frac{1}{p+1} \Phi^{p+1} \right) \tag{2.3}$$

and probably consider general functions $\mathcal{F}$. An equation of motion for field Φ reads

$$\mathcal{F}(\Box)\Phi = \Phi^p. \tag{2.4}$$

Note that ξ does not depend on p or d. Also the limit $p \to 1+$ should be taken with care and for this purpose one cannot fix α'.

Cosmological scenarios built on action (2.3) are given by the following covariantization which accounts a minimal coupling of the tachyon to the gravity

$$S = \int d^d x \sqrt{-g} \left(\frac{R}{2\kappa^2} + \frac{1}{g_o^2} \left(\frac{1}{2} \Phi \mathcal{F}(\Box_g) \Phi - \frac{1}{p+1} \Phi^{p+1}(x) - \Lambda_\Phi - T \right) \right) \tag{2.5}$$

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[4]We always use the signature $(-, +, +, +, \dots)$.

where

$$\Box_g = \frac{1}{\sqrt{-g}} \partial_\mu \sqrt{-g} g^{\mu\nu} \partial_\nu$$

is the Beltrami-Laplace (BL) operator. Here g is a metric, κ is a gravitational coupling constant $\kappa^2 = 8\pi G = \frac{1}{M_P^2}$ and we choose such units that it is dimensionless, T encodes perfect fluids which may be considered including the cosmological constant Λ. Λ_Φ is also a constant but we separate it from the cosmological one. It is considered as a part of a scalar field potential so that in the picture where the scalar field potential has a non-perturbative minimum Λ_Φ cancels its energy. We define for the sequel $m_p^2 \equiv g_0^2 M_P^2$. Though this is obviously not a full theory which may come form open-closed string interactions it is obvious that such a minimal coupling gives a starting point to have an insight into the problem of an open string modes behavior in a curved space-time.

In the present analysis we focus on the four dimensional Universe with a spatially flat FRW metric which can be written as

$$g_{\mu\nu} = \mathrm{diag}(-1, a^2, a^2, a^2) \tag{2.6}$$

with $a = a(t)$ being a space homogeneous scale factor. In this particular case the BL operator is expressed as

$$\Box_g = -\partial_t^2 - 3H\partial_t + \frac{1}{a^2}\partial_{x_i}^2 \tag{2.7}$$

where $H \equiv \dot{a}/a$ is the Hubble parameter and the dot denotes the time derivative. We discuss only a time-dependent scalar field as well. Thus we can think about the BL operator just as

$$\mathcal{D} = -\partial_t^2 - 3H\partial_t.$$

If all the fluids in action (2.5) are coupled one to each other only through the gravity then equations of motion following in this case are

$$3m_p^2 H^2 = \rho_\Phi + \sum_i \rho_i, \tag{2.8a}$$

$$m_p^2(3H^2 + 2\dot{H}) = -(p_\Phi + \sum_i p_i), \tag{2.8b}$$

$$\mathcal{F}(\mathcal{D})\Phi = \Phi^p, \tag{2.8c}$$

$$\dot{\rho}_i + 3H(1 + w_i)\rho_i = 0 \text{ for any } i. \tag{2.8d}$$

Here ρ-s are energies and p-s are pressures of fluids and i enumerates perfect fluids coming from the T-term in the action. ρ_Φ and p_Φ account Λ_Φ. Equations (2.8c) and (2.8d) are consequences of the covariant energy-momentum tensor conservation. So, they are not independent. Since we take w_i being constants equation (2.8d) can be easily solved to give

$$\rho_i = r_i \left(\frac{a_0}{a}\right)^{3(1+w_i)} \text{ and } p_i = w_i \rho_i.$$

r_i and a_0 are constants giving an energy density and the scale factor at some specific time point t_0. One of three remaining equations (2.8a)–(2.8c) is not independent and it will

found convenient in the sequel to work with (2.8c) and the following equation

$$\frac{\ddot{a}}{a} = -\frac{1}{6m_p^2}\left(\rho_\Phi + 3p_\Phi + \sum_i (1 + 3w_i)\rho_i\right). \tag{2.9}$$

Note that w_Φ which is defined through $p_\Phi = w_\Phi \rho_\Phi$ is not a constant.

3. Asymptotic tachyon spectroscopy

Focusing on open string modes we want to emphasize here general facts about a spectrum of Lagrangian (2.3) in an asymptotic regime. We start specifying two desired properties of operator $\mathcal{F}$:

- $\mathcal{F}(z)$ is an analytic function on a complex plane;

- $\mathcal{F}(z)$ admits Taylor expansion at $z = 0$ such that $\mathcal{F}(z) = c_n z^n$ where $c_0 = 1$ and all c_n are real.

These assumptions are very general. Also we restrict ourselves to have $p > 1$ so that there is a non-perturbative vacuum at $\Phi = 1$. In an important case of odd p we also have a symmetry $\Phi \to -\Phi$ so that $\Phi = -1$ also becomes a vacuum. A potential is

$$V = -\frac{1}{2}\Phi^2 + \frac{1}{p+1}\Phi^{p+1} + \Lambda_\Phi. \tag{3.1}$$

In cubic super SFT one gets $p = 3$. The zero value of the potential in the minimum is assured by choosing $\Lambda_\Phi = \frac{p-1}{2(p+1)}$.

The picture we have in mind is a rolling tachyon [41] which starts rolling from an unstable perturbative vacuum $\Phi = 0$ and approaches a non-perturbative one in an infinite time. Thus an asymptotic we are going to study is $\Phi = 1 - \psi$. A linearization around the true vacuum gives the following action [26, 29]

$$S = \frac{1}{g_o^2}\int dx\sqrt{-g}\left(\frac{1}{2}\psi\mathcal{F}(\mathcal{D})\psi - \frac{p}{2}\psi^2\right). \tag{3.2}$$

This can be considered as an effective action in the true vacuum of the SFT. According to the Sen conjectures [41] we expect that there should not be open string excitations. This is simply translated here imposing that operator $\mathcal{F}(\mathcal{D}) - p$ has no zeros for finite ω^2 which are eigenvalues of the BL operator. This is so for approximate operator (2.2) if again $p > 1$.

An approach we use is based on the Weierstrass product method and has been studied in details in the recent paper by I.Ya. Aref'eva and I.V. Volovich [30] (also [44]). Here we just mention main steps.

Any eigenfunction of the BL operator with an eigenvalue ω^2 is an eigenfunction of the $\mathcal{F}$ operator if $\mathcal{F}(\omega^2)$ is well defined. Here we do not specify rigorously a class of functions on which we define an action of $\mathcal{F}$ just mentioning that these are smooth complex valued functions on the real axis with all order derivatives well defined. However, a question of an asymptotic behavior and an integrability is open. For instance, exponentially growing

modes may be physically important if we consider the model from the point of view of cosmological perturbations. Thus

$$\mathcal{D}f_\omega = \omega^2 f_\omega \;\Rightarrow\; \mathcal{F}(\mathcal{D})f_\omega = \mathcal{F}(\omega^2)f_\omega.$$

However, in general $\mathcal{F}(\omega^2)$ may have (infinitely) many branches thus giving that (infinitely) many functions f_ω may correspond to the same eigenvalue of the $\mathcal{F}$ operator. In other words $\mathcal{F}$ may have (infinitely) degenerate eigenvalues and this is a case of operator (2.2) which has all eigenvalues infinitely degenerate [29].

Provided we can solve the characteristic equation

$$\mathcal{F}(\omega^2) = p \tag{3.3}$$

to find all ω_k^2 then it is possible to reformulate asymptotic action (3.2) as an (infinite) sum of noninteracting scalar fields as follows

$$S = \frac{1}{g_o^2} \int dx \sqrt{-g} \frac{1}{2} \sum_k \epsilon_k \psi_k (\mathcal{D} - \omega_k^2)(\mathcal{D} - \omega_k^{2*})\psi_k. \tag{3.4}$$

Here ϵ_k are constants. Recall that equation (3.3) should not have real roots. Thus all ω^2 are complex and for each root ω_k^2 exists a complex conjugate root ω_k^{2*} since $\mathcal{F}(\omega^2)$ can be expressed as a polynomial with real coefficients. Equation of motion looks like

$$(\mathcal{D} - \omega_k^2)(\mathcal{D} - \omega_k^{2*})\psi_k = 0 \tag{3.5}$$

and has a general four parametric solution and these parameters are enough to make ψ_k real. Indeed, solution for ψ_k may be written as

$$\psi_k = \alpha_+ \psi_{k+} + \alpha_- \psi_{k-} + \bar{\alpha}_+ \psi_{k+}^* + \bar{\alpha}_- \psi_{k-}^* \tag{3.6}$$

where $\psi_k^{(i)}$ are two linear independent solutions to the equation $(\mathcal{D} - \omega_k^2)\psi_k = 0$. Obviously complex conjugate functions solve the second order differential equation with a conjugate ω_k^{2*} and all the four terms in the latter expression are linear independent. α-s are integration constants to be adjusted giving real ψ_k. Each such a ψ_k solves an equation of motion coming from action (3.2). Since this equation is linear in ψ then an arbitrary linear combination of ψ_k does solve it.

So, we have reproduced the spectrum by virtue of an (infinite) sum of non-interacting scalar fields. A comment is in place here. One can say that a more simple Lagrangian than (3.4) may be formulated where only first power of the BL operator enters the expression. It is formally possible but since ω_k^2 are complex one would deal in this case with a non-hermitian Lagrangian

$$S = \frac{1}{g_o^2} \int dx \sqrt{-g} \frac{1}{2} \sum_k \left(\varepsilon_k \psi_k (\mathcal{D} - \omega_k^2)\psi_k + \bar{\varepsilon}_k \bar{\psi}_k (\mathcal{D} - \omega_k^{2*})\bar{\psi}_k \right). \tag{3.7}$$

Here ε_k and $\bar{\varepsilon}_k$ are constants. Equations of motion are

$$(\mathcal{D} - \omega_k^2)\psi_k = 0, \quad (\mathcal{D} - \omega_k^{2*})\bar{\psi}_k = 0. \tag{3.8}$$

They have general two parametric solutions

$$\psi_k = \alpha_+ \psi_{k+} + \alpha_- \psi_{k-}, \quad \bar{\psi}_k = \bar{\alpha}_+ \bar{\psi}_{k+} + \bar{\alpha}_- \bar{\psi}_{k-}. \tag{3.9}$$

Constants ε are arbitrary so far. They are fixed by the Weierstrass product to be $\varepsilon_k = \mathcal{F}'(\omega_k^2)$ and $\bar{\varepsilon}_k = \mathcal{F}'(\omega_k^{2*})$. In the next section we will show explicitly that action (3.7) reproduces correctly an energy-momentum tensor of action (3.2) if $\psi = \sum_k \psi_k + \sum_k \bar{\psi}_k$ and $\bar{\psi}_k = \psi_k^*$. Being possible one should use the latter Lagrangian with care and solutions for ψ_k will be necessary complex in order to make full ψ real. It is obvious, that $\bar{\psi} \sim \psi^*$. However, a situation is different compared to action (3.4). A requirement for solution to equation of motion (3.5) to be real is natural and this guaranties a reality of ψ. In case of latter action (3.7) the requirement of a reality of ψ which is $\sum_k \psi_k + \sum_k \bar{\psi}_k$ is an external one. Nevertheless, the latter form of the action is found to be useful. Also we have to say that the above construction becomes more involved in case of multiple roots[5] [30, 44].

4. Cosmology in tachyon vacuum

In this section we are going to cosmological scenarios described by action (2.5). The main goal is to investigate the tachyon near its true vacuum.

4.1 Derivation of cosmological quantities

The energy-momentum tensor is derived by means of

$$T_{\alpha\beta} = -\frac{2}{\sqrt{-g}} \frac{\delta S}{\delta g^{\alpha\beta}}.$$

In these notations $T_{00} = \rho$ and $T_{ii} = a^2 p$. For a general $\mathcal{F}$ the only series expansion in powers of the BL operator may lead to a result. For action (2.5) linearized with $\Phi = 1 - \psi$ one yields

$$\rho_\psi = \frac{K + P}{2}, \quad p_\psi = \frac{K - P}{2} \tag{4.1}$$

$$\text{where } K = \sum_{n=1}^{\infty} c_n \sum_{l=0}^{n-1} \partial_t \mathcal{D}^l \psi \partial_t \mathcal{D}^{n-1-l} \psi, \quad P = \sum_{n=1}^{\infty} c_n \sum_{l=0}^{n-1} \mathcal{D}^l \psi \mathcal{D}^{n-l} \psi$$

where c_n come from $\mathcal{F}(z) = c_n z^n$. One can check that $\nabla_\mu T_\nu^\mu = 0$ on equation of motion (2.4). Expression for the state parameter w_Φ reads

$$w_\Phi = \frac{p_\Phi}{\rho_\Phi} = \frac{K - P}{K + P} = -1 + \frac{2K}{K + P}. \tag{4.2}$$

The total effective equation of state parameter is given by

$$w = \frac{p_\Phi + \sum_i p_i}{\rho_\Phi + \sum_i \rho_i} = \frac{K - P + \sum_i w_i \rho_i}{K + P + \sum_i \rho_i} = -1 - \frac{2}{3} \frac{\dot{H}}{H^2}. \tag{4.3}$$

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[5]I would like to thank S.Yu. Vernov for pointing this out.

Further manipulations can be performed in the latter expressions using $\psi = \sum_n \psi_n$ and representation (3.6) where all the components are eigenfunctions of the BL operator with known eigenvalues. For real ψ we should take

$$\psi = \sum_k \left(\psi_{k+} + \psi_{k+}^* + \psi_{k-} + \psi_{k-}^* \right)$$

where all integration constants α are included in ψ-s and

$$\mathcal{D}\psi = \sum_k \left(\omega_k^2 \psi_{k+} + \omega_k^{2*} \psi_{k+}^* + \omega_k^2 \psi_{k-} + \omega_k^{2*} \psi_{k-}^* \right).$$

Surprisingly the sums over n and l can be evaluated in a closed form to yield the following simple expressions

$$
\begin{aligned}
K &= \sum_k \left(\mathcal{F}'(\omega_k^2) \left(\dot\psi_{k+} + \dot\psi_{k-} \right)^2 + \mathcal{F}'(\omega_k^{2*}) \left(\dot\psi_{k+}^* + \dot\psi_{k-}^* \right)^2 \right), \\
P &= \sum_k \left(\omega_k^2 \mathcal{F}'(\omega_k^2) \left(\psi_{k+} + \psi_{k-} \right)^2 + \omega_k^{2*} \mathcal{F}'(\omega_k^{2*}) \left(\psi_{k+}^* + \psi_{k-}^* \right)^2 \right).
\end{aligned}
\tag{4.4}
$$

Here prime denotes a derivative with respect to an argument. The main achievement at this stage is that all the information can be extracted by means of solving algebraic equation (3.3) and constructing of eigenfunctions of the BL operator. Sum over k is indefinite until a subset of eigenfunctions of interest is not specified.

There are two remarkable properties worth to mention. First, the coefficients can be expressed entirely in terms of function $\mathcal{F}(z)$ without annoying summations. This is a great simplification and an opening for a possibility of studying general operators $\mathcal{F}$. Second, there are no mixed terms involving ψ-s for different k as well as $\psi_i \psi_j^*$ combinations. Thus we have shown explicitly that the energy of scalar field Φ can be derived from action (3.7) if we put $\varepsilon_k = \mathcal{F}'(\omega_k^2)$ and $\bar\varepsilon_k = \mathcal{F}'(\omega_k^{2*})$ and take a special solution $\bar\psi_k = \psi_k^*$.

A mechanism how specific excitations can be selected dynamically is hidden in initial conditions for field Φ at zero time and a correspondence between these conditions and a late time behavior is not completely clear in a curved background.

We stress that all the above consideration in this Subsection does not depend on a background and applicable to a general form of the BL operator.

4.2 Perturbative approach

The relevant construction of the previous section does not rely on a specific background and is valid for a very general operator $\mathcal{F}$ and we want to emphasize the following common points.

First, the spectrum of Lagrangian (3.7) does not depend on a background. Only the form of operator $\mathcal{F}$ determines eigenvalues ω_k^2. Thus, changing a background most probably does not change the spectrum. There may be, however, very special backgrounds where some of eigenvalues ω_k^2 may not have corresponding non-zero eigenfunctions. The latter issue is closely related to a rigorous definition of a class of functions which ψ belongs to.

Second, in a wide range of meaningful functions $H(t)$ the spectrum of the BL operator is the complex plane not restricted in any way unless we do not restrict our physical states. Thus, changing $\mathcal{F}$ without changing a background will shift eigenvalues without major changes to the physics. Hence changing of a background is the most significant modification. This in turn changes the BL operator and its eigenfunctions.

Third, in general knowing eigenfunctions of the BL operator in the flat space we expect an oscillating behavior of ψ since all ω-s are complex. Manipulating with α-s in (3.9) we might be able to construct vanishing or going to wild oscillations solutions. We would not avoid oscillations completely. There is, however, one specific case when for some ω_n^2 oscillations in cosmological quantities will not die or grow. It may be a very interesting situation.

Having the tachyon coupled to a dynamical gravity we have to solve all the system of cosmological equations (2.8). So in general a background is not fixed and subject to dynamical equations. Along scalar field Φ we should find corrections to the metric, i.e. scale factor a in our case. Zero approximation for a denoted by $a^{(0)}$ is an expression given by perfect fluids coming from the T term in action (2.5) only. Indeed, $a^{(0)}$ is the scale factor calculated for $\Phi = 1$. $\Lambda_\Phi = \frac{p-1}{2(p+1)}$ which is aimed to cancel a negative vacuum energy of the scalar field disappears and we are left with term T only. The latter, however, also may contain a constant which we name the cosmological constant. Therefore term T forms what we name a "background" while interaction with the scalar field produces "corrections". The following perturbative strategy can be used:

- We take $a = a^{(0)}$ and solve an equation of motion for the scalar field which is now (3.8);

- We substitute resulting $\Phi = 1 - \sum_n \psi_n$ in (2.9) and find correction $a^{(1)}$.

Equation (2.9) linearized around $a^{(0)}$ becomes

$$-\ddot{a}^{(1)} + v(t)a^{(1)} = \frac{a^{(0)}}{6m_p^2}(\rho_\psi + 3p_\psi) = \frac{a^{(0)}}{6m_p^2}(2K - P) \qquad (4.5)$$

where

$$v(t) = \frac{\ddot{a}^{(0)}}{a^{(0)}} + \frac{1}{2m_p^2}\sum_i (1 + w_i)(1 + 3w_i)r_i \left(\frac{a_0}{a^{(0)}}\right)^{3(1+w_i)}.$$

This procedure can be repeated with found first order corrections to find next order perturbations. This is beyond of our present analysis but one comment is very important. In general this may bring out corrections of order ψ^2 indicating thereby that a linear approximation around the true vacuum is no longer valid and higher corrections should be accounted in the action.

5. Tachyon in initially flat space

We start with the most simple example. Namely, we take $T = 0$ meaning that only the tachyon does present and assume that it is in the asymptotic regime. Such a situation may

be if during the evolution of the tachyon up to some time a back reaction of the gravity is negligible. Also for simplicity we take only one mode in ψ. In this case taking $\psi = \alpha + i\beta$, $\omega^2 = m^2 + ik^2$, $\mathcal{F}'(\omega^2) = x + iy$ and assuming $\bar{\psi} = \psi^*$ we have the following action for fields α and β

$$S = \frac{1}{g_o^2} \int dx \sqrt{-g} \left(\alpha(x\mathcal{D} - xm^2 + yk^2)\alpha - \beta(x\mathcal{D} - xm^2 + yk^2)\beta - \right. \tag{5.1}$$
$$\left. -2\alpha(y\mathcal{D} - ym^2 - xk^2)\beta \right).$$

We see, that for any signs of parameters one normal and one phantom field present in the system. This is in accord with Ostrogradski method [45, 46] (see [30] for a discussion on this point). Note, that only field α is physical one since $\psi + \psi^* = 2\alpha$ is a fluctuation around the minimum of the tachyon potential. Constants m^2 and k^2 are certainly real but not necessarily positive. Moreover, $k^2 \neq 0$. Thus, the dynamics of the tachyon near the vacuum is governed effectively by two scalar fields one of which is a phantom. The action may serve as a toy model for the tachyon around its vacuum. The energy and pressure associated with the above Lagrangian are as follows

$$\rho = x\dot{\alpha}^2 - x\dot{\beta}^2 + 2y\dot{\alpha}\dot{\beta} + (xm^2 - yk^2)(\alpha^2 - \beta^2) - 2(ym^2 + xk^2)\alpha\beta,$$
$$p = x\dot{\alpha}^2 - x\dot{\beta}^2 + 2y\dot{\alpha}\dot{\beta} - (xm^2 - yk^2)(\alpha^2 - \beta^2) + 2(ym^2 + xk^2)\alpha\beta. \tag{5.2}$$

Friedmann equations for the above action minimally coupled to the gravity read

$$3m_p^2 H^2 = \rho,$$
$$m_p^2 \dot{H} = -x\dot{\alpha}^2 + x\dot{\beta}^2 - 2y\dot{\alpha}\dot{\beta}. \tag{5.3}$$

Plus to this two equations of motion for scalar fields are

$$(x\mathcal{D} - xm^2 + yk^2)\alpha - (y\mathcal{D} - ym^2 - xk^2)\beta = 0,$$
$$(x\mathcal{D} - xm^2 + yk^2)\beta + (y\mathcal{D} - ym^2 - xk^2)\alpha = 0. \tag{5.4}$$

Only three of four above equations are independent. The limit we are interesting in is α and all its derivatives go to zero, so the tachyon rests in its vacuum. However, according to equations of motion (5.4) $\alpha = 0$ leads to $\beta = 0$ for any real values of parameters. This is not a desired result demonstrating that provided $T = 0$ the tachyon itself (plus Λ_Φ which makes its potential energy non-negative) cannot generate any non-trivial cosmology. A possible scenario that α oscillates near zero and β makes the job of a cosmology generation does not work. Although there is no analytic prove, the conclusion has been checked numerically in a wide range of parameters.

However, once we do not require vanishing or bounded field α various possibilities for the cosmological evolution appear. It may be of great importance to understand whether growing solutions for α can be physically justified and moreover a mechanism which suppresses such a growing at some time scale is needed.

Alternatively we can add some ingredient to the dynamics so that the evolution becomes interesting. Two important examples are developed in the next section.

6. Two more examples

Below we analyze specific background configurations of perfect fluids and different possibilities for eigenvalues ω_k^2 and intensively use obtained formulae (4.4) where the summation has been done for a general operator $\mathcal{F}$.

6.1 Tachyon with cosmological constant

In paper [29] operator (2.2) and $p = 3$ situation was analyzed in a background of the cosmological constant. The first order corrections were found in this model. Here we relax $p = 3$ condition and do not assume any exact form of operator $\mathcal{F}$.

Zero approximation for the scale factor is

$$a^{(0)} = a_0 e^{H_0 t} = a_0 e^{\sqrt{\frac{\Lambda}{3m_p^2}} t}. \tag{6.1}$$

An equation for eigenfunctions of the BL operator reads

$$\ddot{\psi}_k + 3H_0 \dot{\psi}_k = -\omega_k^2 \psi_k$$

and the solution is

$$\psi_{k+} = \alpha_{k+} e^{-\omega_{k+}t}, \quad \psi_{k-} = \alpha_{k-} e^{-\omega_{k-}t},$$

$$\text{where } \omega_{k\pm} = \frac{3}{2}H_0 \pm \sqrt{\frac{9}{4}H_0^2 - \omega_k^2}, \quad \alpha_{k\pm} \text{ are constants.} \tag{6.2}$$

Equation (4.5) becomes

$$-\ddot{a}^{(1)} + H_0^2 a^{(1)} = \frac{e^{H_0 t}}{6m_p^2}(2K - P). \tag{6.3}$$

The solution is

$$a^{(1)} = a_+ e^{H_0 t} + a_- e^{-H_0 t} + \bar{a}(t) \tag{6.4}$$

where the last term is a particular solution to the inhomogeneous equation. If for any non-zero ψ_{k+} one takes $\psi_{k-} = 0$ and vice versa then K and P completely free of mixed terms. Therefore, since equation (6.3) is linear we can solve it for only one specific mode and then sum up the answers. Hence, for an only mode the result for the scalar field is

$$\Phi = 1 - \alpha e^{-rt}\cos(\nu t + \varphi) \tag{6.5}$$

where $\alpha = 2|\alpha_{k\pm}|$ and r and ν are real and imaginary parts for $\omega_{k\pm} = r_{k\pm} + i\nu_{k\pm}$ with k fixed and a sign chosen. We assume $r \geq 0$ so that oscillations do not grow.[6] The r.h.s. of equation (6.3) can be represented as

$$\frac{a_0 e^{(H_0 - 2r)t}}{6m_p^2}(2\alpha_K \sin(2\nu t + \varphi_K) - \alpha_P \sin(2\nu t + \varphi_P)).$$

---

[6]For positive r one easily deduces the results of [29].

Solution for $\bar{a}(t)$ can be found to be

$$
\bar{a}(t) = \frac{e^{(H_0 - 2r)t}}{24 m_p^2 (r^2 + \nu^2)((H_0 - r)^2 + \nu^2)} \times \tag{6.6}
$$
$$
\times \left((\nu^2 - r^2 + H_0 r)(2\alpha_K \sin(2\nu t + \varphi_K) - \alpha_P \sin(2\nu t + \varphi_P)) + \right.
$$
$$
\left. + (H_0 - 2r)\nu(2\alpha_K \cos(2\nu t + \varphi_K) - \alpha_P \cos(2\nu t + \varphi_P)) \right).
$$

So, correction $a^{(1)}$ is given by (6.4) and (6.7). Now one can readily get resulting expressions for H and the state parameter. It is interesting that playing with r and ν one can observe different behaviors. For instance, for $r = H_0/2$ oscillations in $a(t)$ will not die despite the fact that oscillations in Φ vanish. On the other hand it is impossible to avoid oscillations completely. These oscillations will be translated to w_Φ and effective total state parameter w making possible periodic transition between phantom and quintessence phases.

Thus we have shown explicitly that a periodic crossing of the phantom divide may occur as well as a possibility of non-vanishing oscillations of the cosmological quantities even if the scalar field tends to its vacuum.

6.2 Tachyon with perfect fluid $w > -1$

Title dictates the following equation

$$
3 m_p^2 H^{(0)2} = r \left(\frac{a_0}{a} \right)^{3(1+w)} \tag{6.7}
$$

which can be solved since $H = \dot{a}/a$. Results for $a^{(0)}(t)$ and $H^{(0)}(t)$ are

$$
a^{(0)}(t) = a_0 \left(\frac{t - t_s}{t_0 - t_s} \right)^{\frac{2}{3(1+w)}}, \quad H^{(0)}(t) = \frac{2}{3(1+w)} \frac{1}{t - t_s} \tag{6.8}
$$

where t_s is a constant. An equation for eigenfunctions of the BL operator reads

$$
\ddot{\psi}_k + \frac{2}{(1+w)} \frac{1}{t - t_s} \dot{\psi}_k = -\omega_k^2 \psi_k.
$$

For arbitrary parameters a solution may be written in terms of Kummer special functions. However, in the large t limit the following asymptotic behavior remains

$$
\psi_{k+} = \alpha_{k+} t^{-\frac{1}{1+w}} e^{i\omega_k t}, \quad \psi_{k-} = \alpha_{k-} t^{-\frac{1}{1+w}} e^{-i\omega_k t},
$$
$$
\dot{\psi}_{k+} = \left(i\omega_k - \frac{1}{(1+w)t} \right) \psi_{k+}, \quad \dot{\psi}_{k-} = \left(-i\omega_k - \frac{1}{(1+w)t} \right) \psi_{k-}. \tag{6.9}
$$

To analyze the solution we use $\omega_k = r_k + i\nu_k$. Depending on the sign of ν_k either exponentially growing or suppressed oscillations will be in the dynamics of the scalar field. On top of this $t^{-\frac{1}{1+w}}$ factor always persists. Thus even if $\nu_k = 0$ the oscillations will vanish. Result for a single mode in the scalar field is as follows

$$
\Phi = 1 - \alpha t^{-\frac{1}{1+w}} e^{-\nu t} \cos(rt + \varphi). \tag{6.10}
$$

Here $\alpha = 2|\alpha_{k\pm}|$, r and ν are r_k and ν_k for a fixed k. Equation (4.5) reads

$$-\ddot{a}^{(1)} + \frac{v_0}{(t-t_s)^2}a^{(1)} = A(t-t_s)^{\frac{2}{3(1+w)}}(2K - P) \qquad (6.11)$$

where $v_0 = (1+3w)\left(-\frac{1}{9(1+w)^2} + (1+w)r(t_0-t_s)^2\right)$ and $A = \frac{a_0}{6m_p^2}(t_0-t_s)^{-\frac{2}{3(1+w)}}$. v_0 is not obviously positive or negative. A solution for $a^{(1)}$ is

$$a_+t^{\frac{1}{2}+\sqrt{\frac{1}{4}+v_0}} + a_-t^{\frac{1}{2}-\sqrt{\frac{1}{4}+v_0}} + \bar{a}(t) \quad \text{for } v_0 > -\frac{1}{4},$$

$$a_+\sqrt{t} + a_-\sqrt{t}\log t + \bar{a}(t) \quad \text{for } v_0 = -\frac{1}{4}, \quad (6.12)$$

$$a_+\sqrt{t}\cos\left(\sqrt{-v_0-\frac{1}{4}}\log t\right) + a_-\sqrt{t}\sin\left(\sqrt{-v_0-\frac{1}{4}}\log t\right) + \bar{a}(t) \quad \text{for } v_0 < -\frac{1}{4}$$

where t_s constant is neglected. The last term is a particular solution to the inhomogeneous equation. The leading asymptotic in the r.h.s. of equation (6.11) can be represented as

$$A\alpha_P t^{-\frac{4}{3(1+w)}}e^{-2\nu t}\sin(2rt + \varphi_P).$$

Solution for $\bar{a}(t)$ can be found but the final expression is very cumbersome even if one fixes w. Numeric calculations are helpful to prove that for any v_0 correction $a^{(1)}$ becomes to be small and vanishing oscillations. The only assumption made is that we take a vanishing solution for ψ. Thus the system looks like a shaking around a background formed by a perfect fluid.

7. Discussion and further directions

To conclude we summarize the main results achieved above.

Non-local action (2.1) with a general operator $\mathcal{F}$ is analyzed and a local formulation for a linearization near a non-perturbative vacuum is given by (3.7). It does rely on roots of characteristic equation (3.3) only. The energy and pressure are formulated for a general function $\mathcal{F}(z)$ without specifying its explicit form. Moreover, this analysis does not depend on a background.

A model example where interaction with gravity is turned on in the final stage of the tachyon evolution is considered. It is demonstrated that just a rolling tachyon is not capable to generate a non-trivial cosmology. A natural question how other components may enter the action immediately arises.

Two more examples where the scalar field is accompanied by the cosmological constant or by a perfect fluid are explored in details. It is shown, in particular, that our scalar field generates a crossing of the phantom divide in the cosmological constant background. This crossing is periodic one and moreover, a condition of non-vanishing oscillations is formulated. The Big Rip singularity problem is avoided because w exhibits a non-trivial time dependence and consequently is not a constant less then -1.

It would be very interesting to continue the lines of the present analysis and try to formulate a local action in case of a full non-linear model. If possible, this will open a

way to make use of numeric methods in analyzing the model. One natural question to be answered in this way whether the early time dynamics may generate some cosmic fluids which decouple at the late stage of the tachyon evolution. This will justify an appearance of an additional term T in the action at large times.

Also it would be interesting to invert the problem and see how different forms of operator $\mathcal{F}$ and the potential affect the cosmology. This may shade light on a structure of an effective tachyon action if higher excitations are taken into account in the SFT.

Another generalization is an inclusion of closed string scalar fields, the closed string tachyon and dilaton in the analysis to understand a role of closed string excitations and probably explain the T term.[7]

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